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Anomaly cancellation in noncritical string theory

Jaume Gomis

Perimeter Institute for Theoretical Physics
Waterloo, Ontario N2L 2Y5, Canada
E-mail: jgomis@perimeterinstitute.ca

Abstract: We construct new two dimensional unoriented superstring theories in two dimensions with a chiral closed string spectrum and show that anomalies cancel upon supplying the appropriate chiral open string degrees of freedom imposed by tadpole cancellation.

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1. Introduction and conclusion

The “miraculous” cancellation of anomalies in ten dimensional superstring theory [1] gave rise for the first time to a consistent framework that incorporates quantum gravity and chiral gauge couplings akin to the ones in the standard model. By now, a beautiful picture has emerged of how potential anomalies can cancel upon compactification from ten dimensions.

In this note we construct new two dimensional non-critical superstring theories with parity violating couplings and show that anomalies can be cancelled. These low dimensional theories are simple toy models of string theory which nevertheless capture – in a calculable setting – important concepts and techniques of more realistic string vacua. These include D-branes, RR fluxes, holography, dualities and as we show in this note cancellation of gauge and gravitational anomalies.

We construct these theories by performing suitable orientifold projections of the known two dimensional noncritical superstring theories. It is shown that theories for which the orientifold plane carries RR charge have an anomalous closed string spectrum. Nevertheless, we show that imposing RR tadpole cancellation — a basic worldsheet consistency condition [2, 3] — introduces the correct chiral (open) string degrees of freedom to cancel all anomalies.

Some of these models have an unusual particle spectrum, consisting only of spacetime fermions, which makes it challenging to find a matrix model formulation of these theories; there are also no known unstable D-branes that can be used to define the closed string vacuum via tachyon condensation [4]. Recently, in [5], an M-theory formulation of noncritical string theory has been proposed. An important consistency check of that proposal is to identify the string models found in this note in the three dimensional M-theory construction.

The plan of the rest of the note is as follows. In section 2 we summarize the known noncritical superstring theories in two dimensions, study their symmetries and classify the possible orientifold projections. In section 3 the constraints imposed by anomaly cancellation and tadpole cancellation are presented. Section 4 shows that the anomalies due to the chiral closed string spectrum of the orientifold models are cancelled by the open string degrees of freedom induced by tadpole cancellation.

2. Two dimensional noncritical superstring theories

In this section we summarize the known superstring theories[1] in two non-compact dimensions, their massless spectrum and corresponding symmetries.[2] We then classify the admissible orientifold projections, which give rise to new unoriented string theories in two dimensions. The cancellation of anomalies and tadpoles in these theories will be studied in the rest of the note.

Just like in ten dimensions, there are two consistent "type II" [8−10] GSO projections:[3]

$$\begin{array}{cc} \underline{\text{Type IIB}} & \underline{\text{Type IIA}} \\ & \\ (NS+,NS+) & (NS+,NS+) \\ (NS-,R-)\rightarrow\Psi_- & (NS-,R+)\rightarrow\Psi_+ \\ (R-,NS-)\rightarrow\tilde{\Psi}_- & (R-,NS-)\rightarrow\Psi_- \\ (R+,R+)\rightarrow C_+ & (R+,R-). \end{array} \tag{2.1}$$

The level matching condition forces the GSO projection in the (NS,R) and (R,NS) sectors to be *different* than in ten dimensions.

The spectrum of particles of the type IIB string is a left moving boson C_+ and two right moving Majorana-Weyl fermions Ψ_- and $\tilde{\Psi}_-$, while the type IIA string has a left and right moving Majorana-Weyl fermion denoted by Ψ_+ and Ψ_- respectively.

The discrete symmetry group of two dimensional type IIB string theory is the dihedral group of four elements $\mathbf{D}_4$, generated[4] by $a=\Omega(-1)^F$ and $b=\Omega$, where Ω stands for the worldsheet parity operator and $(-1)^{F(\bar{F})}$ denotes the left(right)-moving spacetime fermion number operator. The complete list of elements of the symmetry group is $G=\{1,\Omega,(-1)^F,(-1)^{\bar{F}},(-1)^{F+\bar{F}},\Omega(-1)^F,\Omega(-1)^{\bar{F}},\Omega(-1)^{F+\bar{F}}\}$. The type IIA discrete symmetry group is $\mathbb{Z}_2\times\mathbb{Z}_2$, generated by $(-1)^F$ and $(-1)^{\bar{F}}$. As in ten dimensions, the theory is not invariant[5] under the action of Ω.

[1]That is theories with $\mathcal{N}=(1,1)$ supersymmetry on the worldsheet. There are also heterotic theories with $\mathcal{N}=(1,0)$ supersymmetry [6, 7].

[2]We assume throughout that we are in the pure linear dilaton vaccuum, i.e. we do not turn on any of the possible marginal deformations, which may break some of the symmetries.

[3]One can get isomorphic copies of these theories by the action of space-time parity, which reverses the sign of the GSO projection in all R sectors.

[4]The relations in the group are $a^4=b^2=1$ and $(ab)^2=1$.

[5]If we relax the condition of translational invariance along the x coordinate in which the dilaton does not vary, the symmetry group enhances to the dihedral group, which is now generated by $a=\Omega\mathrm{R}(-1)^F$ and $b=\Omega\mathrm{R}$, where R is a space-time parity transformation acting by $x\rightarrow -x$.

There are two consistent type 0 [11, 12] GSO projections:

$$\begin{array}{cc} \underline{\text{Type 0B}} & \underline{\text{Type 0A}} \\ & \\ (NS+, NS+) & (NS+, NS+) \\ (NS-, NS-) \to T & (NS-, NS-) \to T \\ (R-, R-) \to C_- & (R-, R+) \\ (R+, R+) \to C_+ & (R+, R-). \end{array} \tag{2.2}$$

Therefore, the spectrum of particles of the type 0B string is two scalars T and $C = C_+ + C_-$, while the type 0A string has a single scalar field T.

The symmetry group of type 0B string theory is $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, generated by $\Omega, (-1)^F$ and $(-1)^f$, where $(-1)^{f(\bar{f})}$ denotes the left(right)-moving worldsheet fermion number operator. Type 0A string theory has $\mathbf{D}_4$ as its[6] group of symmetries, and is generated by $a = \Omega(-1)^f$ and $b = \Omega$.

Now that we have found the symmetries of the various theories, we are ready to construct new unoriented superstring theories. The unoriented theories we construct are obtained by modding out a string theory by the orientifold group $G = \{1, \Omega g\}$, where Ωg is a symmetry generated by the combined action of the worldsheet parity operator Ω and a discrete symmetry generator g. In order for the orientifold to be well defined we must find group elements g such that $(\Omega g)^2 = 1$ when acting on the string spectrum.[7] We are thus lead to the following orientifold projections:

Type IIB orientifolds

$$\begin{aligned} G_1 &= \{1, \Omega\} \\ G_2 &= \{1, \Omega(-1)^{F+\bar{F}}\}\,. \end{aligned}$$

Type 0B orientifolds

$$\begin{aligned} G_1 &= \{1, \Omega\} \\ G_2 &= \{1, \Omega(-1)^F\} \\ G_3 &= \{1, \Omega(-1)^f\}\,. \end{aligned} \tag{2.3}$$

Type 0A orientifolds

$$G_1 = \{1, \Omega\}\,.$$

Of these orientifolds, we study here the type IIB orientifolds and the type 0B string modded out by G_3, which suffer from potential anomalies and RR tadpoles. The rest of the

[6] If we allow for broken translational invariance along x, the type 0A string is invariant under the action of R.

[7] If $(\Omega g)^2 \neq 1$, we can still construct a consistent orientifold group, but it is of the type $\tilde{G} = H_1 \cup \Omega H_2$. This can be interpreted as an orientifold of an orbifold by H_1, but modding by H_1 yields another theory in the list, whose orientifolds we already considered. For example, in type IIB, $(\Omega(-1)^F)^2 = (-1)^{F+\bar{F}}$ so that modding out by $\tilde{G}$ yields in this case an orbifold of type IIB by $(-1)^{F+\bar{F}}$ which is type 0B, so that we recover a type 0B orientifold.

models — which have no anomalies nor RR tadpoles — have already been studied in [13] (see also [14]) in the context of a non-perturbative description in tems of a matrix model.[8] It would be very interesting to find a matrix model description of the models studied here.

3. Anomalies and tadpoles

The cancellation of anomalies [15, 1] is an important constraint on a string vaccum, which has to be diagnosed in our two dimensional superstring theory constructions. In two dimensions, a right handed Majorana-Weyl fermion transforming in an n-dimensional representation ρ of the gauge group G yields the following anomaly polynomial

$$I_{1/2}(\rho) = \frac{n}{96}\,\mathrm{tr}\,R^2 - \frac{1}{4}\mathrm{tr}_\rho F^2\,, \tag{3.1}$$

while a right moving chiral boson contributes:

$$I_C = \frac{1}{48}\,\mathrm{tr}\,R^2\,. \tag{3.2}$$

The spectrum of type IIB in two dimensions is chiral, but as mentioned in [8] anomalies cancel:

$$I_{\mathrm{IIB}} = 2\times I_{1/2}(\rho=1) - I_C = 0\,. \tag{3.3}$$

In the following we compute the closed string spectrum of the unoriented string theories constructed by modding out by the orientifold groups presented in the previous section. We shall see that the closed string spectrum is anomalous. Cancellation of RR tadpoles, however, introduces precisely the right chiral open string degrees of freedom to cancel anomalies. In this section we present the formulas that are needed to compute RR tadpoles, which we are going to apply to the specific models in the rest of the note.

The computation of the tadpole due to the orientifold plane is easiest to perform by evaluating the Klein bottle vacuum diagram. The tadpole can be obtained by factorizing the loop diagram into the closed tree channel, which encodes the one point function responsible for the tadpole.

In the two dimensional context under consideration, all particles are massless and the computation of the Klein bottle is easiest to perform as a sum over the target space *closed* string spectrum

$$Z_{KB}(\Omega g) = \frac{V}{2}\int\frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}e^{-\pi t p^2}\sum_{S\in\mathcal{H}_c}(-1)^{\mathbf{Fs}}(\Omega g)_S\,, \tag{3.4}$$

where $(-1)^{\mathbf{Fs}} = \pm 1$ is the space-time fermion number of the state S and $(\Omega g)_S = \pm 1$ is the eigenvalue of the operator Ωg on the state S. The tadpole can be extracted by factorizing to the tree channel

$$Z_{KB}(\Omega g) = \frac{V}{8\pi^3}\int_0^\infty ds\sum_{S\in\mathcal{H}_c}(-1)^{\mathbf{Fs}}(\Omega g)_S\,, \tag{3.5}$$

where $s = \pi/2t$ for the Klein bottle.

[8]Type 0B modded out by G_3 was not considered in [13] since the super-Lioville interaction breaks the $(-1)^f$ symmetry. Here we consider the pure dilaton vacuum state, for which $(-1)^f$ is a symmetry.

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Whenever an orientifold plane carries charge under a RR 2-form field C_2, the tadpole may be cancelled by adding D1-branes charged under C_2. Whether tadpole cancellation can be accomplished hinges on whether the contribution from the Klein bottle can be cancelled by the contribution from the Möbius strip and cylinder diagram. These can be computed by summing over the *open* string spectrum on the D1-branes. The cylinder diagram on N such D-branes is given by

$$Z_C(\Omega g) = N^2 \frac{V}{2} \int \frac{dt}{2t} \int \frac{d^2p}{(2\pi)^2} e^{-2\pi t p^2} \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} , \tag{3.6}$$

which yields in the tree channel

$$Z_C(\Omega g) = N^2 \frac{V}{32\pi^3} \int_0^\infty ds \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} , \tag{3.7}$$

where $s = \pi/t$ for the cylinder.

In order to compute the last diagram, the Möbius strip, we recall how an orientifold group element acts on an open string state $|S\rangle$. This state is represented by an $N \times N$ matrix S and the orientifold projection $G = \{1, \Omega g\}$ keeps the state satisfying

$$S = (\Omega g)_S \gamma_{\Omega g} S^T \gamma_{\Omega g}^{-1} , \tag{3.8}$$

where $(\Omega g)_S = \pm 1$ is the eigenvalue of Ωg on the state $|S\rangle$ and S^T denotes the transpose of the matrix S. A universal open string mode is a gauge field A for which $(\Omega g)_A = -1$ always so that consistency allows for two possible gauge groups:

$$\begin{aligned} \gamma_{\Omega g} &= \gamma_{\Omega g}^T \rightarrow \ \mathrm{SO}(N) \text{ gauge group} \\ \gamma_{\Omega g} &= -\gamma_{\Omega g}^T \rightarrow \ \mathrm{Sp}(N/2) \text{ gauge group} . \end{aligned} \tag{3.9}$$

The eigenvalue $(\Omega g)_S$ for the state $|S\rangle$ determines the representation ρ of the gauge group under which S transforms.

The Möbius strip is then given by

$$Z_M(\Omega g) = \mathrm{Tr}(\gamma_{\Omega g}^{-1}\gamma_{\Omega g}^T) \frac{V}{2} \int \frac{dt}{2t} \int \frac{d^2p}{(2\pi)^2} e^{-2\pi t p^2} \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} (\Omega g)_S , \tag{3.10}$$

which gives in the tree channel

$$Z_M(\Omega g) = \mathrm{Tr}(\gamma_{\Omega g}^{-1}\gamma_{\Omega g}^T) \frac{V}{8\pi^3} \int_0^\infty ds \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} (\Omega g)_S , \tag{3.11}$$

where $s = \pi/4t$ for the Möbius strip.

Therefore, tadpole cancellation in the orientifold by $G = \{1, \Omega g\}$ requires that:

$$N^2 \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} + 4 \sum_{S\in\mathcal{H}_c} (-1)^{\mathbf{F_S}} (\Omega g)_S + 4\mathrm{Tr}(\gamma_\Omega^{-1}\gamma_\Omega^T) \sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{F_S}} (\Omega g)_S = 0 . \tag{3.12}$$

We now show for the various new models defined in section 2 that tadpoles can be cancelled and that the precise chiral degrees of freedom needed to cancel anomalies arise in the open string sector.

4. New unoriented noncritical strings

Having already gathered the relevant formulas for anomalies and tadpoles in the previous section, we now just require computing the action of Ωg on the various closed and open string states.

The basic symmetry we mod out by is worldsheet parity, generated by Ω. In order to fully characterize the models, we must specify the action of Ω on worldsheet fermions, spin fields and the closed string vacuum state. It is given by:

$$
\begin{aligned}
\Omega: \qquad & \begin{array}{ll}\psi & \to \bar{\psi} \\ \bar{\psi} & \to -\psi\end{array} \Longrightarrow \Omega : \bar{\psi}\psi \to \bar{\psi}\psi \\
\Omega: \qquad & \begin{array}{ll}S^\alpha & \to \bar{S}^\alpha \\ \bar{S}^\alpha & \to S^\alpha\end{array} \Longrightarrow \Omega : \bar{S}^\alpha \otimes S^\beta \to -\bar{S}^\beta \otimes S^\alpha \\
& \Omega \cdot |0\rangle_{NS-NS} = |0\rangle_{NS-NS} \,.
\end{aligned} \tag{4.1}
$$

We now study the various string theories.

4.1 Modding type IIB by $G_1 = \{1, \Omega\}$

Given the action of Ω in (4.1) it follows that the only state surviving the projection is a right-moving Majorana-Weyl fermion Ψ_-^S, where

$$
\Psi_-^S \equiv \Psi_- + \tilde{\Psi}_- \qquad \Psi_-^A \equiv \Psi_- - \tilde{\Psi}_- \,, \tag{4.2}
$$

while Ψ_-^A and C_+ are projected out.

The spectrum of the model, as it stands, is anomalous. Extra degrees of freedom must be added to the system to cancel gravitational anomalies. We now show that the introduction of the required degrees of freedom follows by imposing cancellation of RR tadpoles.

The Klein bottle diagram (3.5) is non-vanishing since

$$
\sum_{S \in \mathcal{H}_c} (-1)^{\mathbf{F_S}} (\Omega)_S = -1 + 1 - 1 = -1 \,, \tag{4.3}
$$

and the O1-plane carries RR charge and the model is inconsistent. We can cure this by adding suitable D1-branes such that the total RR charge vanishes.

We need to construct the boundary state corresponding to a D1-brane in two dimensional type IIB string theory. The corresponding boundary state is given by

$$
|D1\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_{NSNS} - |-\rangle_{NSNS}\right) + \frac{1}{\sqrt{2}} \left(|+\rangle_{RR} - |-\rangle_{RR}\right) , \tag{4.4}
$$

which yields the following GSO projection and spectrum in the open string channel

$$
\begin{array}{ccc}
 & \underline{\mathrm{D1-brane}} & \\
(NS+) & \to & A \\
(R+) & \to & \lambda_+ \,,
\end{array} \tag{4.5}
$$

where A is a non-dynamical gauge field and λ_+ is a right moving Majorana-Weyl spinor transforming in the representation ρ of the gauge group G.

Using the fact that

$$(\Omega)_{\lambda_+} = -1\,, \tag{4.6}$$

we find that λ_+ transforms in the same way as the gauge field, i.e. in the ρ = adjoint representation of the gauge group G. We can now compute the cylinder (3.7) and Möbius strip (3.11) contribution to the tadpole:

$$\begin{aligned}\sum_{S\in\mathcal{H}_o}(-1)^{\mathbf{F_S}} &= -1\\ \sum_{S\in\mathcal{H}_o}(-1)^{\mathbf{F_S}}(\Omega)_S &= -1\cdot -1 = 1\,.\end{aligned} \tag{4.7}$$

Tadpole cancellation (3.12) requires that

$$N^2 + 4 - 4\,\mathrm{Tr}(\gamma_\Omega^{-1}\gamma_\Omega^T) = 0\,, \tag{4.8}$$

which implies that $N = 2$ and that $\gamma_\Omega = \gamma_\Omega^T$. Therefore, the open string gauge group is $G = \mathrm{SO}(2)$ and λ_+ transforms in the ρ=adjoint representation of SO(2), which for SO(2) is the trivial $\rho = 1$ representation.

Summarizing, the spectrum of the model is:

Spectrum of type IIB/$\{1,\Omega\}$

- Closed: Ψ_-^S
- Open: SO(2) gauge field A; λ_+ in $\rho = 1$ rep. of SO(2)

The fermion structure coming from the open string sector and which follows from tadpole cancellation makes the model free of gravitational and gauge anomalies.

4.2 Modding type IIB by $G_1 = \{1, \Omega(-1)^{F+\bar{F}}\}$

Given the action of Ω in (4.1) and the usual action of $(-1)^{F,(\bar{F})}$ (which acts by -1 on the corresponding left(right) moving spin field) it follows that the only state surviving the projection is the right-moving Majorana-Weyl fermion Ψ_-^A, while Ψ_-^S and C_+ are projected out. The closed string spectrum of this model is the same as the previous one, and is therefore anomalous.

The Klein bottle diagram (3.5) is non-vanishing and is the same as before

$$\sum_{S\in\mathcal{H}_c}(-1)^{\mathbf{F_S}}(\Omega)_S = -1 + 1 - 1 = -1\,, \tag{4.9}$$

and the O1-plane carries RR charge and the model is inconsistent. We can cure this by adding suitable D1-branes such that the total RR charge vanishes.

The cylinder diagram (3.7) is also the same as before

$$\sum_{S\in\mathcal{H}_o}(-1)^{\mathbf{F_S}} = -1\,, \tag{4.10}$$

so that the open string spectrum is a gauge field A and a right moving Majorana-Weyl spinor λ_+ transforming in the representation ρ of the gauge group G.

Using the fact that

$$(\Omega(-1)^{F+\bar{F}})_{\lambda_+} = +1\,, \tag{4.11}$$

we find that λ_+ transforms differently than the gauge field; λ_+ transforms[9] in the $\rho =$ symmetric(antisymmetric) representation of $G = \mathrm{SO}(N)(\mathrm{Sp}(N/2))$.

The Möbius strip diagram (3.11) is different since $(-1)^{F+\bar{F}}$ acts non-trivially on λ_+

$$\sum_{S\in\mathcal{H}_o} (-1)^{\mathbf{Fs}} (\Omega(-1)^{F+\bar{F}})_S = -1\cdot 1 = -1\,. \tag{4.12}$$

Tadpole cancellation (3.12) in this model requires that

$$N^2 + 4 + 4\,\mathrm{Tr}(\gamma_\Omega^{-1}\gamma_\Omega^T) = 0\,, \tag{4.13}$$

which implies that $N = 2$ and that $\gamma_\Omega = -\gamma_\Omega^T$. Therefore, the open string gauge group is $G = \mathrm{Sp}(1) \simeq \mathrm{SU}(2)$ and λ_+ transforms in the $\rho = \begin{smallmatrix}\square\\\square\end{smallmatrix}$ representation of $\mathrm{Sp}(1)$ which for $\mathrm{Sp}(1)$ is the trivial $\rho = 1$ representation.

Summarizing, the spectrum of the model is:

Spectrum of type $\mathrm{IIB}/\{1, \Omega(-1)^{F+\bar{F}}\}$

- Closed: Ψ_-^A
- Open: $\mathrm{Sp}(1)$ gauge field A; λ_+ in $\rho = 1$ rep. of $\mathrm{Sp}(1)$.

The fermion structure coming from the open string sector and which follows from tadpole cancellation makes the model free of gravitational and gauge anomalies.

4.3 Modding type 0B by $G_1 = \{1, \Omega(-1)^f\}$

Given the GSO projection of type 0B string theory (2.2) and the action of Ω in (4.1) it follows that the right moving scalar C_- is the only state state surviving the orientifold projection while T and C_+ are projected out. Since C_- is a chiral field in two dimensions, the model is anomalous.

We now show that also in this model the constraint imposed by tadpole cancellation cancels all anomalies.

The Klein bottle diagram (3.5) is given by

$$\sum_{S\in\mathcal{H}_c} (-1)^{\mathbf{Fs}} (\Omega)_S = -1 - 1 + 1 = -1\,, \tag{4.14}$$

and the O1-plane carries RR charge and the model is inconsistent. We can cure this by adding suitable D1-branes such that the total RR charge vanishes.

It is well known that in type 0B string theory (before orientifolding) there are two different D1-branes, whose boundary states are given by:

$$\begin{aligned} |D1,+\rangle &= |+\rangle_{NSNS} + |+\rangle_{RR} \\ |D1,-\rangle &= |-\rangle_{NSNS} + |-\rangle_{RR}\,. \end{aligned} \tag{4.15}$$

[9] For $G = \mathrm{SO}(N)(\mathrm{Sp}(N/2))$ the adjoint is the antisymmetric(symmetric) representation.

The action of $\Omega(-1)^f$ exchanges $|D1,+\rangle$ with $|D1,-\rangle$. Therefore in order to be able to mode out by our symmetry $\Omega(-1)^f$, we must have the same number N of $|D1,+\rangle$ branes as $|D1,-\rangle$ branes.

Correspondingly, there are three sectors of open strings corresponding to $D1_+ - D1_+$ strings, $D1_- - D1_-$ strings and $D1_+ - D1_-$ strings (the $D1_- - D1_+$ strings are the complex conjugate of the $D1_+ - D1_-$ strings). The boundary state overlaps produce the following GSO projection and spectrum in the open string channel:

$$\frac{D1_+ - D1_+}{(NS,+) \to A_1} \qquad \frac{D1_- - D1_-}{(NS,+) \to A_2} \qquad \frac{D1_+ - D1_-}{(R,+) \to \lambda_+} \tag{4.16}$$

A_1 and A_2 are $\mathrm{U}(N)$ gauge fields while λ_+ is a right moving *complex* Weyl fermion transforming under the bifundamental representation of $\mathrm{U}(N) \times \mathrm{U}(N)$.

After the orientifold projection we are left with a $\mathrm{U}(N)$ gauge field A, since the two gauge groups above get identified by the action of $\Omega(-1)^f$. Furthermore, using that

$$(\Omega(-1)^f)_{\lambda_+} = -1\,, \tag{4.17}$$

we find that λ_+ transforms in $\rho = \begin{smallmatrix}\square\\\square\end{smallmatrix}$ representation of $\mathrm{U}(N)$. The cylinder (3.7) and Möbius strip (3.11) are given by:

$$\begin{aligned}\sum_{S\in\mathcal{H}_o}(-1)^{\mathbf{F_S}} &= -1\\ \sum_{S\in\mathcal{H}_o}(-1)^{\mathbf{F_S}}(\Omega(-1)^f)_S &= -1\cdot -1 = 1\,.\end{aligned} \tag{4.18}$$

Tadpole cancellation (3.12) then requires that

$$N^2 + 4 - 4N = 0\,, \tag{4.19}$$

which implies that $N = 2$. Therefore, the open string gauge group is $G = \mathrm{U}(2)$ and λ_+ transforms in $\rho = \begin{smallmatrix}\square\\\square\end{smallmatrix}$ representation of $\mathrm{U}(2)$, which for $\mathrm{U}(2)$ is the $\rho = 1$ trivial representation.

Summarizing, the spectrum of the model is:

Spectrum of type $0\mathrm{B}/\{1, \Omega(-1)^f\}$

- Closed: C_-
- Open: $\mathrm{U}(2)$ gauge field A; complex λ_+ in $\rho = 1$ rep. of $\mathrm{U}(2)$

The fermion structure coming from the open string sector and which follows from tadpole cancellation makes the model free of gravitational and gauge anomalies since the anomaly polynomial vanishes $I_C - 2 \times I_{1/2}(\rho = 1) = 0$, as shown in (3.3).

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